

# Section 2.3

## GRAPHS OF POLYNOMIAL FUNCTIONS

Make a chart:

|                              | 1 <sup>linear</sup><br>$x^1$ | 2 <sup>quadratic</sup><br>$x^2$ | 3 <sup>cubic</sup><br>$x^3$ | 4 <sup>quartic</sup><br>$x^4$ | 5 <sup>quintic</sup><br>$x^5$ |
|------------------------------|------------------------------|---------------------------------|-----------------------------|-------------------------------|-------------------------------|
| Degree                       | 1                            | 2                               | 3                           | 4                             | 5                             |
| # of "turns"                 | 0                            | 1                               | 2                           | 3                             | 4                             |
| Positive Leading Coefficient |                              |                                 |                             |                               |                               |
| Negative Leading Coefficient |                              |                                 |                             |                               |                               |

Conclusions:

# of Turns: one less than the degree

**ODD DEGREE**

opposite end behavior:

one arm is up.

one is down

**EVEN DEGREE**

same end behavior:

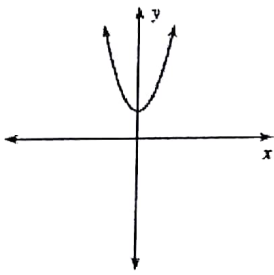
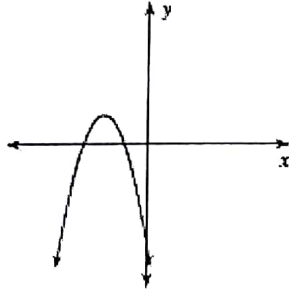
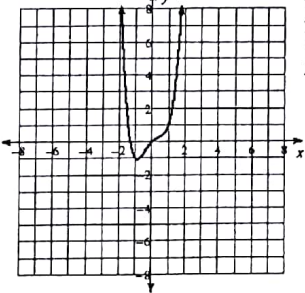
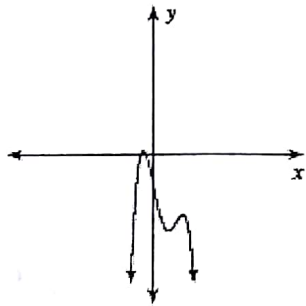
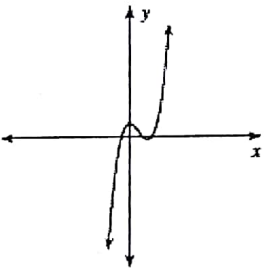
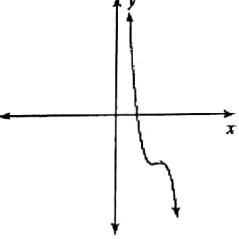
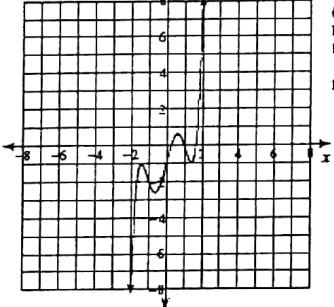
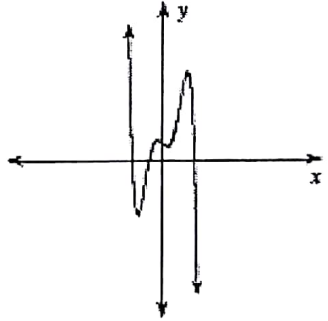
arms are going

the same direction

|             | Positive L.C.  | Negative L.C.   |
|-------------|--|---|
| Odd Degree  | $\lim_{x \rightarrow -\infty} f(x) \rightarrow -\infty$ $\lim_{x \rightarrow +\infty} f(x) \rightarrow \infty$ | $\lim_{x \rightarrow -\infty} f(x) \rightarrow \infty$ $\lim_{x \rightarrow +\infty} f(x) \rightarrow -\infty$  |
| Even Degree | $\lim_{x \rightarrow -\infty} f(x) \rightarrow \infty$ $\lim_{x \rightarrow +\infty} f(x) \rightarrow \infty$  | $\lim_{x \rightarrow -\infty} f(x) \rightarrow -\infty$ $\lim_{x \rightarrow +\infty} f(x) \rightarrow -\infty$ |

Describe the end behavior and the number of turns for each graph below:

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|  |   |   |   |
|--|---|---|---|
| <p>Ex 1:<br/><math>f(x) = x^2 + 2</math></p>  <p><math>\lim_{x \rightarrow -\infty} f(x) = \infty</math><br/><math>\lim_{x \rightarrow \infty} f(x) = \infty</math></p>           | <p>Ex 2:<br/><math>f(x) = -x^2 - 6x - 7</math></p>  <p><math>\lim_{x \rightarrow -\infty} f(x) = -\infty</math><br/><math>\lim_{x \rightarrow \infty} f(x) = -\infty</math></p>            | <p>Ex 3:<br/><math>f(x) = x^4 - x^2 + x</math></p>  <p><math>\lim_{x \rightarrow -\infty} f(x) = \infty</math><br/><math>\lim_{x \rightarrow \infty} f(x) = \infty</math></p>             | <p>Ex 4:<br/><math>f(x) = -x^4 + 3x^3 - 2 - 5x</math></p>  <p><math>\lim_{x \rightarrow -\infty} f(x) = -\infty</math><br/><math>\lim_{x \rightarrow \infty} f(x) = -\infty</math></p> |
| <p>Ex 5:<br/><math>f(x) = x^3 - 2x^2 + 1</math></p>  <p><math>\lim_{x \rightarrow -\infty} f(x) = -\infty</math><br/><math>\lim_{x \rightarrow \infty} f(x) = \infty</math></p> | <p>Ex 6:<br/><math>f(x) = -x^3 + 10x^2 - 33x + 32</math></p>  <p><math>\lim_{x \rightarrow -\infty} f(x) = \infty</math><br/><math>\lim_{x \rightarrow \infty} f(x) = -\infty</math></p> | <p>Ex 6:<br/>5) <math>f(x) = x^5 - 4x^3 + 4x - 1</math></p>  <p><math>\lim_{x \rightarrow -\infty} f(x) = -\infty</math><br/><math>\lim_{x \rightarrow \infty} f(x) = \infty</math></p> | <p>Ex 8<br/><math>f(x) = -x^5 + 4x^3 - x + 1</math></p>  <p><math>\lim_{x \rightarrow -\infty} f(x) = \infty</math><br/><math>\lim_{x \rightarrow \infty} f(x) = -\infty</math></p>  |

What can you generalize about the end behavior and its function?

What can you generalize about the number of turns and its function?

# Sketching a Graph of Polynomials

The following information is needed for sketching a graph of any polynomials:

Odd or even degree

End behavior

How many turns (degree - 1)

Ex 9:  $f(x) = -x^2 - 6x - 7$

even degree

$\lim_{x \rightarrow -\infty} f(x) = -\infty$

$\lim_{x \rightarrow \infty} f(x) = -\infty$

$x \rightarrow \infty$

1 turn



Ex 10:  $f(x) = x^3 - 2x^2 + 1$

odd degree

$\lim_{x \rightarrow -\infty} f(x) = -\infty$     $\lim_{x \rightarrow \infty} f(x) = \infty$

2 turns



Ex 11:  $f(x) = -x^5 + 4x^3 - x + 1$

odd degree

$\lim_{x \rightarrow -\infty} f(x) = \infty$

$\lim_{x \rightarrow \infty} f(x) = -\infty$

4 turns



Ex 12:  $f(x) = x^4 - 3x^3 - 2 - 5x$

Even degree

$\lim_{x \rightarrow -\infty} f(x) = \infty$

$\lim_{x \rightarrow \infty} f(x) = \infty$

3 turns



Ex 13:  $f(x) = x^4 + 8x^3 + 4x^2 + 2$

Even degree

$\lim_{x \rightarrow -\infty} f(x) = \infty$

$\lim_{x \rightarrow \infty} f(x) = \infty$

3 turns



Ex 14:  $f(x) = x^3 + 3x^2 - x - 3$

odd degree

$\lim_{x \rightarrow -\infty} f(x) = -\infty$

$\lim_{x \rightarrow \infty} f(x) = \infty$

2 turns



Ex 15:  $f(x) = x^2 - 8x + 2$

Even degree

$\lim_{x \rightarrow -\infty} f(x) = \infty$

$\lim_{x \rightarrow \infty} f(x) = \infty$

1 turn



$x \rightarrow \infty$

2 turns

factored form.

$$16) f(x) = (x-3)(5-x)(2x-7)$$

Sketch

By hand (include x-int(s))

$$x-3=0$$

$$x=3$$
  
 $(3,0)$

$$5-x=0$$

$$x=5$$
  
 $(5,0)$

$$2x-7=0$$

$$x=\frac{7}{2}$$
  
 $(3.5,0)$

Describe

end behavior

$$\lim_{x \rightarrow -\infty} f(x) = \infty$$

$$\lim_{x \rightarrow \infty} f(x) = -\infty$$

leading term:  $(x)(-x)(2x)$

$$\rightarrow -2x^3$$

$$(-3)(5)(-7)$$
  
 $(0, 105)$

